The velocity distribution being parabolic, the ventilation capacity becomes:

$$V = \frac{2}{3}\, b_c\,(h_c/2)\, v_{max} = \frac{1}{3} A_c v_{max} \qquad (4.54)$$

By using the energy equation (4.1) you get:

$$\Delta T = \frac{Q_s}{\rho_o\, c_p V} = \frac{3\, Q_s}{\rho_o\, c_p\, v_{max} A_c} \qquad (4.55)$$

which can be used to eliminate ΔT in eq. (4.53). Together with eq. (4.14a) and by inserting the values of the constants you get successively:

$$v_{max} = \left[\frac{3\, Q_s\, g\, R\, h_c}{c_p\, p_o\, C_d\, A}\right]^{1/3} \left[\frac{T_o}{T_i}\right]^{1/3} \left[\frac{1}{\psi}\right]^{1/2} = 0{,}043 \left[\frac{Q_s\, h_c}{C_d\, A}\right]^{1/3} \left[\frac{1}{\psi}\right]^{1/2} \qquad (4.56)$$

$$\Delta T = \left[\frac{3\, Q_s R}{c_p p_o\; C_d A}\right]^{2/3} \left[\frac{1}{g\; h_c}\right]^{1/3} \left[\frac{T_o}{T_i}\right]^{1/3} T_o$$

$$= 2.0 \cdot 10^{-4}\, T_o \left[\frac{Q_s}{C_d\, A}\right]^{2/3} \left[\frac{1}{h_c}\right]^{1/3} \qquad (4.57)$$

$$V = \frac{1}{3} A_c \cdot 0.043 \left[\frac{Q_s\, h_c}{C_d\, A}\right]^{1/3} \left[\frac{1}{\psi}\right]^{1/2} = 0.014\, (C_d\, A)^{2/3}\, (Q_s\, h_c)^{1/3} \qquad (4.58)$$

The requirement to the opening dimensions to get a certain ventilation capacity or a certain temperature difference can then be determined by, when assuming $h_c \simeq 0.8$ h:

$$b^{2/3} h = 79.4\; V/(Q_s^{1/3}\;\; C_d^{2/3}) \qquad (4.59)$$

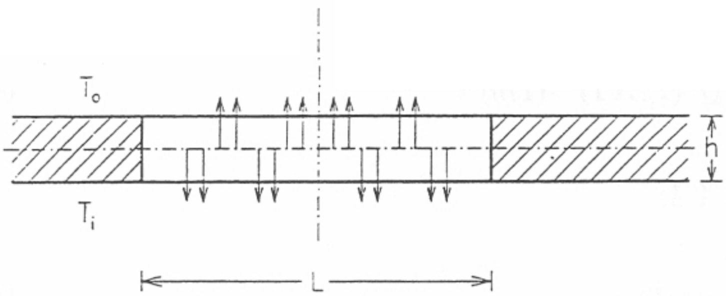


Figure 4.10 *Air flow through one horizontal opening*

By using the energy equation (4.1) these expressions can be turned into:

$$V = (0.18\,A)^{2/3}\left[\frac{Q_s\,g\,h}{c_p\,\rho_o\,T_i}\right]^{1/3} = (0.18\,A)^{2/3}\left[\frac{Q_s\,R\,g\,h}{c_p\,p_o}\right]^{1/3}\left[\frac{T_i}{T_o}\right]^{1/3}$$

$$= 0.0098\,A^{2/3}\,(Q_s\,h)^{1/3} \qquad \textit{for} \quad 0{,}1 < h/A^{1/2} < 0{,}7 \qquad (4.65)$$

$$V = (0.06\,A)^{2/3}\left[\frac{Q_s\,g\,A^{1/2}}{C_p\,\rho_o\,T_i}\right]^{1/3} = (0.06\,A)^{2/3}\left[\frac{Q_s\,R\,g\,A^{1/2}}{c_p\,p_o}\right]^{1/3}\left[\frac{T_i}{T_o}\right]^{1/3}$$

$$= 0.0047\,A^{5/6}\,Q_s^{1/3} \qquad \textit{for} \quad h/A^{1/2} < 0{,}1 \qquad (4.66)$$

By combining (4.63) with (4.65) and (4.64) with (4.66) you get:

$$\Delta T = 3.0 \cdot 10^{-4}\,(Q_s/A)^{2/3}\,T_i\,(1/h)^{1/3} \qquad \textit{for} \quad 0{,}1 < h/A^{1/2} < 0{,}7 \qquad (4.67)$$

$$\Delta T = 6.2 \cdot 10^{-4}\,Q_s^{2/3}\,T_i\,(1/A)^{5/6} \qquad \textit{for} \quad h/A^{1/2} < 0{,}1 \qquad (4.68)$$

From eqs. (4.65) - (4.68) it is finally possible to determine the required opening area as to get a certain ventilation capacity or temperature difference, respectively, and you get for $0{,}1 < h/A^{1/2} < 0{,}7$:

$$\Delta p = \psi \rho v_c^2/2$$

or

$$v_c = ((f_o - f_i)/\psi)^{1/2} v_w \tag{4.76}$$

4.3.1 Wind Velocities and Pressure Coefficients

The wind velocity depends on the height above terrain and the terrain type, and it can be expressed by, cf. figure 4.11:

$$v_w = K v_{10} \ln (h/h_o) \tag{4.77}$$

where h_o is the roughness factor indicating the height above terrain where the wind velocity still can be considered to be zero. The constant K is dependent on x_o, because $K \ln(10/h_o) = 1.0$.

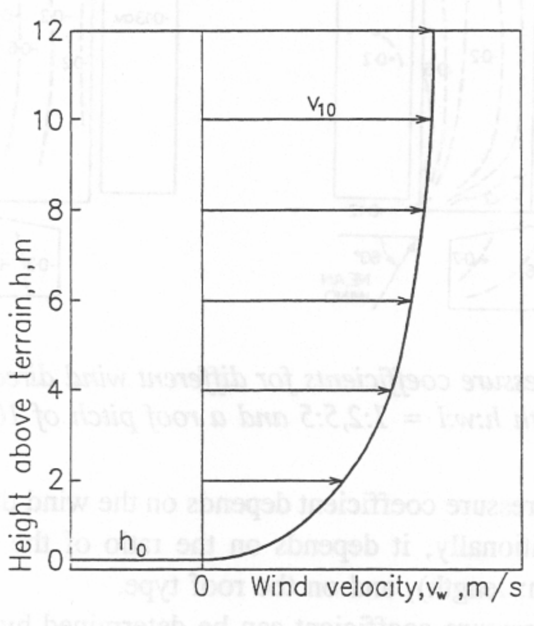
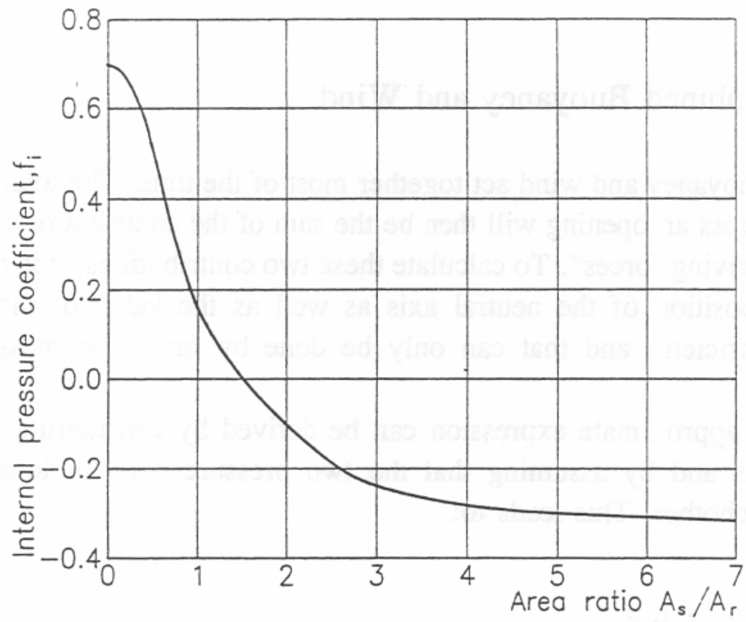


Figure 4.11 Wind speed in dependence of the height above terrain.

or by using eq. (4.76) and assuming that $\rho_i \simeq \rho_o$:

$$\sum ((f_{or} - f_i)/\psi_r)^{1/2} A_{cr} = \sum ((f_i - f_{os})/\psi_s)^{1/2} A_{cs} \tag{4.79}$$

On figure 4.13 are shown calculated values for the internal pressure coefficient in dependence of the area ratio A_s/A_r and when assuming an external pressure coefficient of 0.75 on windward side and of -0.35 on leeward side of the building.



4.13 The internal pressure coefficient as function of the ratio between the outlet and the inlet area, A_s/A_r

4.3.2 Ventilation Capacity

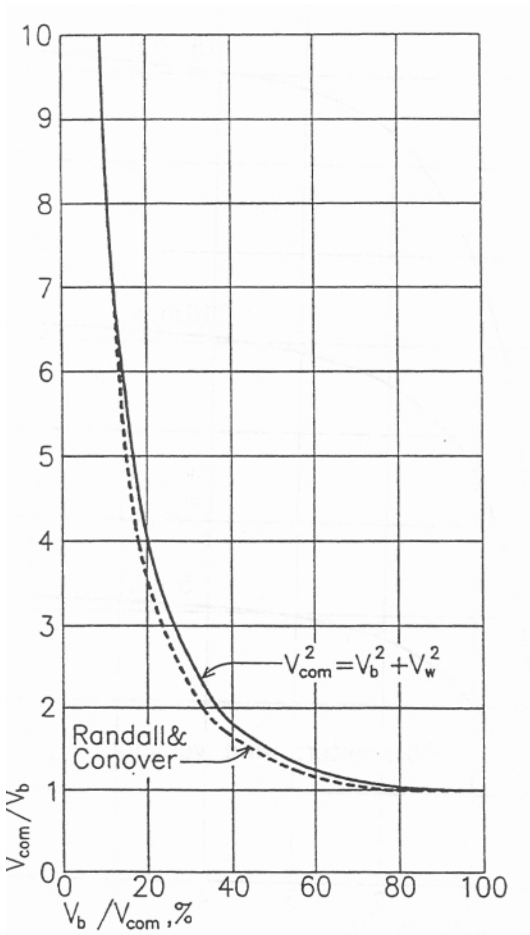
The ventilation capacity can be found as:

$$V = \sum A_{cr} v_r = \sum A_{cr} ((f_{or} - f_i)/\psi_r)^{1/2} v_w$$
$$(\tag{4.80}$$
$$= \sum C_{dr}) A_r (f_{or} - f_i)^{1/2} v_w$$

Because the wind ventilation capacity is independent of the temperature, it may

The expression is valid with good accuracy when either the buoyancy or the wind is dominating, and this will always be the case in an atrium design situation.

On figure 4.14 is shown the comparison between eq. (4.83) and the mean value of the calculation results found by Randall and Conover (1931) on several buildings under different conditions. Taking all their results into consideration you find, that the error by using the eq. (4.83) is biggest, reaching about 20-25%, when the two contributions are almost equal. From a design point of view, this is in fact not critical, as the error can be adjusted by the control system.



4.14 Comparison of eq. (4.83) with the mean curve for the calculations done by Randall and Conover (1931).

4.3.4 Control Possibilities

The internal pressure coefficient is strongly dependent on the distribution of the opening areas on the windward and leeward side of the building. If the openings are placed mainly on the windward side the internal pressure coefficient will the leeward gable and an outward flow through the openings near the windward gable. This may create draft problems if the control system is not able to cope with this situation. A possibility is to make the openings controllable section-wise along the building.

4.4 Infiltration

Air infiltration is the uncontrolled air exchange through cracks and other leakages in the building envelope. The air flow is caused by the pressure differences between inside and outside created by the temperature differences (i.e. thermal buoyancy) and by the wind.

4.4.1 Theoretical Considerations

The flow through the cracks may be laminar or turbulent depending on the crack width and the air velocity . By narrow cracks or low air velocity (by small pressure differences) the flow is laminar and the air flow through the cracks can be determined by the Poiseuille law:

$$V_{crack,l} = \alpha_{lam}\, \Delta p \tag{4.83}$$

where α_{lam} is a flow coefficient which includes the crack dimensions and the viscosity of the air. For wider cracks or higher velocities, the flow will be turbulent and you have:

$$V_{crack,t} = \alpha_{tur}\, \Delta p^{1/2} \tag{4.84}$$

The total air flow through the cracks will depend on the pressure differences as well as the distribution of the cracks on the building envelope. It is not possible to know the crack dimensions and their distribution beforehand and therefore the infiltration is frequently expressed by:

$$V_{infil} = \alpha_{tur}\, \Delta p^{\beta} \tag{4.85}$$

where the exponent ß has a value of 0.5 or 1.0. Measurements indicates that ß has a value of 0.6-0.75 for a long range of Δp (Blomsterberg 1990) or 13 ~ 2/3. In practice it is usual to measure the tightness of a building by a certain pressure difference $\Delta p_{ref.}$ By any other pressure difference, you then have

Table 4.1 Airtightness measured in atria at an overpressure of 50 Pa. (Wall and Blomsterberg, 1994)

Building	Completed, year	ach by 50 Pa	Surrounding buil-dings
Kabi Pharmacia	1991	1.1	Offices
Scandinavian Center	1991	0.8	Offices/shops
Siriushuset	1992	1.0	Offices
Oncological Clinic	1992	0.7	Wards, hos-pital
Piggvaren	1986	14	Residential[1]
Tärnan	1983	8.8	Residential[2]

(Blomsterberg, 1993)
[2] (Wall, 1992)

4.5 Use of formula. Implementation

The set of formulas can be used in the following ways:

- Directly in the design of the natural ventilation system and for analysis purposes for instance in connections with the design of the control system.
- Implemented into computer programs for thermal simulation of buildings.
- Determination of certain starting values for calculations with CFD-programs (Computational Fluid Dynamics).

It is first of all the formulas for thermal buoyancy which are of interest as the extreme design situations occur by calm weather. A survey of formulas to be used under such conditions are shown in table 4.2 based either on the density difference, on the temperature difference or on the net heat input.

The wind can be coupled through formulas (4.82) and (4.83). The wind will act as a supplement to the thermal buoyancy driven ventilation. The combination of thermal buoyancy and wind ventilation will influence the design of the control system.

4.5.1 Direct Use

The design situation, where the required maximum opening areas are to be determined, occurs in calm summer weather. Then the sufficient ventilation capacity has to be obtained by thermal buoyancy alone. If a required temperature difference not to be exceeded is known, the net heat input can be determined by the heat balance equation and then the required opening areas can be found by eq.(4.22). If you know the air exchange to be obtained, eq. (4.21) can be used.

A minimum ventilation is required in winter when a certain amount of air pollution or moisture has to be removed. This removal requires a certain minimum ventilation capacity. Besides, a certain indoor temperature has to be obtained and then the net heat input can be determined. Finally, the minimum opening areas can be determined by eq. (4.21) and these areas are of interest in connection with the design of the control system.

The formulas in table 4.2 are also suitable for analysis purposes, for instance the air velocities in the openings in connection with comfort considerations under various conditions, or the opening areas in connection with designing the control system.

4.5.2 Implementation in Thermal Simulation Programs

The thermal simulation programs developed to date attach the greatest importance on the indoor climate during winter and on the energy consumption in that connection. The exchange of fresh air is therefore dealt with in a very simple manner, as it can be seen in the description in chapter 9 of some existing used simulation programs, and that is usually sufficient under winter conditions.

In summer natural ventilation is a cooling measure and a high ventilation capacity may be needed. The ventilation capacity varies strongly with opening geometry, opening position and with heat load, and no model for this variation is found as an integrated part of any simulation program. Some programs (like FRES and TRNSYS, see chapter 9) use precalculated data which are introduced into the program as a variable. Other programs (like tsbi3, see chapter 9) use a simple formula for the air change rate per hour depending on temperature difference and wind velocity and including 2-3 constants to be assumed by the program user. But the user or the designer gets no help for determining the necessary opening areas.

The formula (4.22) would be very useful as a subroutine in the simulation programs for determination of the needed opening areas. If the temperature difference ΔT not to be exceeded is known, then also the indoor temperature $T_i = T_o + \Delta T$ will be known, and the net heat input can be taken from the heat balance in the program. The subroutine can step by step follow the calculations and will give the needed opening areas stepwise, so that the maximum value can be determined. If, for instance, a certain maximum opening area should not be exceeded from a structural or another point of view the consequences can be determined by the simulation program.

4.6 Summary

Natural ventilation can be used as a mean to cool atria and the adjacent building. Both buoyancy and wind cause natural ventilation effects. This chapter provide formulas to study natural ventilation by buoyancy and wind separate and together. It also gives data on infiltration in existing and new atria.

Thermal buoyancy can be calculated for two separate, a single vertical, and a single horizontal opening, respectively. For two separate openings, neutral axis and air velocities and ventilation capacity as a function of opening area can be studied. Required and optimum opening area can be calculated. The influence of thermal stratification on natural ventilation is also shown. Resistance and contraction values for the openings are suggested.

Calculation of natural ventilation can be performed by hand or as a part of a simulation in a CFD-program (Computational Fluid Dynamics) or a building energy simulation program.

Suffices

b = buoyancy
c = contracted
d = discharge
= indoor
o = outdoor
io = indoor value at neutral plane level
w = wind
1 = inlet (by two openings)
2 = outlet (by two openings)
r = inlet (by several openings)
s = outlet (by several openings)

5. Surface Heat Transfer Coefficients

5.1 Introduction

Heat is transferred between a building surface and its surroundings by radiation and convection. These two modes of heat transfer act independently of each other, and by the temperatures relevant in connection with building surfaces, the heat transfer per unit surface area can be expressed by:

$$Q/A_s = h_r\,(T_s - T_{a1}) + h_c\,(T_s - T_{a2}) \qquad (5.1a)$$

or, if $T_{a1} = T_{a2} = T_a$:

$$Q/A_s = (h_r + h_c)\,(T_s - T_a) = h(T_s - T_a) \qquad (5.1b)$$

where:

h is the surface heat transfer coefficient

h_r is the radiation heat transfer coefficient

h_c is the convection heat transfer coefficient

A_s is the area of the surface in question

T_s is the temperature of the surface

T_{a1}, T_{a2}, T_a are reference temperatures for the surroundings.

The surface heat transfer coefficient is dependent on temperatures as well as on air velocities close to the surface. For normal insulated building surfaces

$$\Delta Q_r / Q_r = ((\Delta T_{12}/2)/T_m)^2 \tag{5.5}$$

so that the error is less than 1% if ΔT_{12} is smaller than 50 K.

Figure 5.1 shows the correct value of the factor $h_r/\sigma\epsilon_{12}$ (sometimes calle the temperature factor and equal to $(T_1^4 - T_2^4)/(T_1 - T_2)$) as a function of the mean temperature (or the temperature level) and with the temperature difference as a parameter. It can be seen that even for reasonable big temperature ranges the factor is almost independent of Δt. Besides, it is almost linear, and this linearity can be found by expanding T. in the following way:

$$T_m^3 = (T_o + T_m - T_o)^3 = T_o^3(1 + (T_m - T_o)/T_o^3) \sim T_o^3(1 + 3(T_m - T_o)/T_o)$$

where T_0 is a fixed temperature in the middle of the linearity range. The coefficient h_r can thus in a certain range be expressed by:

$$h_r \sim 4\epsilon_{12}\sigma T_o^3 \ (1 + 3(T_m - T_o)/T_o) \tag{5.6}$$

The radiant heat exchange between building surfaces and their surroundings can often be considered as taking place between the surfaces in a so-called enclosure, where one of the surfaces is plane or convex (i.e. it can not "see" any parts of itself) and is totally surrounded by the other surface as shown on Figure 5.2. In this case the effective emissivity can be determined by (Wong, 1977):

$$1/\epsilon_{12} = 1/\epsilon_1 + (A_1/A_2)((1/\epsilon_2) - 1) \tag{5.7}$$

Some relevant emissivity values are shown in Table 5.1. It can be seen that the emissivities of building surfaces usually have a value of 0.9-0.95, unless they are painted or cladded with something bright metallic.

5.2.1 Interior Radiation

Inside the building, the radiant heat exchange takes place between the outer and the inner walls. Unless one of the inner walls is heated for instance by the sun, it can be assumed that the surface temperature of all inner walls is equal to the inside air temperature. You then get following expression for the heat exchange:

$$Q_{ri} = h_{ri}(T_i - T_s)A_1 \quad (5.8)$$

where A_1 is the area of the outer wall surface, T_i is the inside air temperature, and T_s is the outer wall surface temperature.

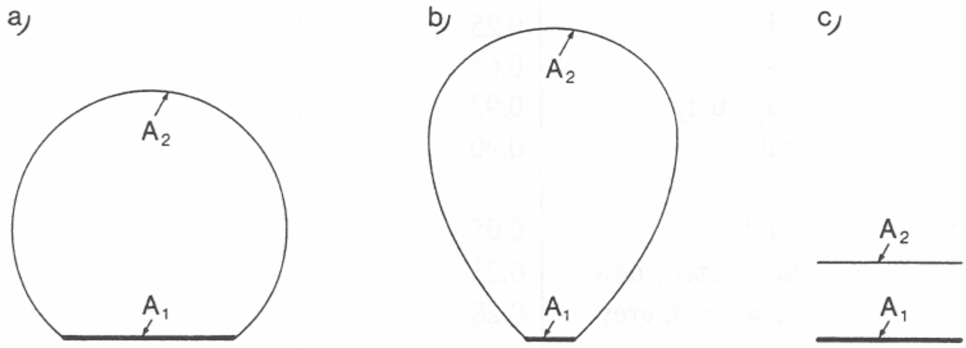


Figure 5.2 Positions of two radiant surfaces.

If the room has only one outer wall, its surface is part of an enclosure for which the effective emissivity can be found from eq. (5.7). For $\epsilon_2 \sim \epsilon_1$ you get:

$$\epsilon_{12} = \epsilon_1/(1 + (A_1/A_2)\ (1-\epsilon_1)) \quad (5.9)$$

Further, with only one outer wall you have that $A_1/A_2 < 1/5$ and you get:

$$\epsilon_{12} \sim \epsilon_1 \quad (5.10)$$

with an error less than 2%, and h_{ri} can then be determined as:

$$h_{ri} = 4\,\epsilon_1\,\sigma\,T_m^3 \quad (5.11)$$

Under normal conditions, T_m will have a value of 283 K - 303 K. In this range and for T_o = 293 K, you get from eq. (5.6):

$$h_{ri} = 4 \cdot \epsilon_1 \cdot 5.67 \cdot 10^{-8} \cdot 293^3\,(1+3\,(T_m-293)/293)$$
$$= 5.7 \cdot \epsilon_1\,(1+0{,}010\,(T_m-293)) \quad (5.12)$$

It can further be seen, that a constant value:

$$h_{ri} = 5.7 \cdot \epsilon_1 \tag{5.13}$$

can be used in the range 288 K - 298 K with an error less than 5-7%, and in the range 283 K - 303 K with an error less than 10-12%.

If one of the inner walls is heated by the sun, equation (5.8) can be used, but now with A_1 as the area of the heated surface, and T_i as the temperature of the remaining surfaces (including the outer wall surface) and assumed to be equal to the inside air temperature. Besides, the radiation heat transfer coefficient can be considered as constant similar to eq.(5.13), but with an about 15% higher value due to the higher temperature level, cf. figure 5.1.

For two outer wall surfaces forming an angle you have to distinguish between if the angle is convex or concave seen from the inside. By a convex angle, eq. (5.8) can be used with A_1 as the total area of the two surfaces and with h_r as a constant value similar to eq.(5.13) but adjusted to the temperature level. If the two outer surfaces form a concave angle each of the outer surfaces has to be treated separately with its own view factor, which can be found in a relevant textbook.

5.2.2 Exterior Radiation

The exterior heat exchange takes place between the exterior building surfaces and the sky, the ground, and the surrounding buildings and vegetation. The heat exchange can be expressed by:

$$Q_{ro} = h_{ro} \, (T_s - T_{eq}) \, A_s \tag{5.14a}$$

where h_{ro} is the exterior radiation heat transfer coefficient, T_S is the temperature of the exterior surface in question with the surface area A_S, and finally T_{eq} is an equivalent temperature representing the surroundings.

5.2.2.1 Sky Temperature. When calculating the radiant heat exchange with the sky, a so-called sky temperature is often introduced in order to simplify the calculations. From a thermal radiation point of view, the sky is then considered as a hemispherical, black surface with a temperature T_{sky}, which gives a radiation equal to the actual measured radiation T , i.e.:

content of the air and thereby the dew point temperature is almost constant during 24 hours.

The sky temperature can be found by inserting the expressions for ε_{sky} into eq. (5.14d). This is done for the equations (5.14e) - (5.14g) in the outdoor air temperature range -20°C to 10°C with a relative humidity of 90%. The resulting sky temperatures are shown on figure 5.3 together with some sky temperatures based on sky emissivities found by other authors.

The measurements behind the sky temperatures on figure 5.3 are carried out on the country side far from towns except for Brown (1956) and Berdahl and Fromberg (1982). The figure shows higher sky temperatures for areas close to towns than on the country side. This is understandable from the point of view, that increasing air pollution increases the sky emissivity. It indicates that is might be reasonable at to distinguish between country side, and town areas, when it concerns sky temperatures. For the country side you get the following linear approximate relationship between sky temperature and outdoor air temperature with the last mentioned being in the range -20°C to 10°C:

$$t_{sky} = -24 + 1{,}2\,t_o \tag{5.15a}$$

or

$$T_{sky} - 273 = -24 + 1{,}2\,(T_o - 273) \tag{5.15b}$$

or

$$T_{sky} = -79 + 1{,}2\,T_o \tag{5.15c}$$

For town areas, figure 5.3 indicates the following relationship:

$$t_{sky} = -14 + 1{,}2\,t_o \tag{5.16a}$$

or

$$T_{sky} = -69 + 1{,}2\,T_o \tag{5.16b}$$

For totally cloudy skies the equivalent radiant temperature becomes much closer to the ambient temperature as shown on Figure 5.4. In this case you get the following approximate expression:

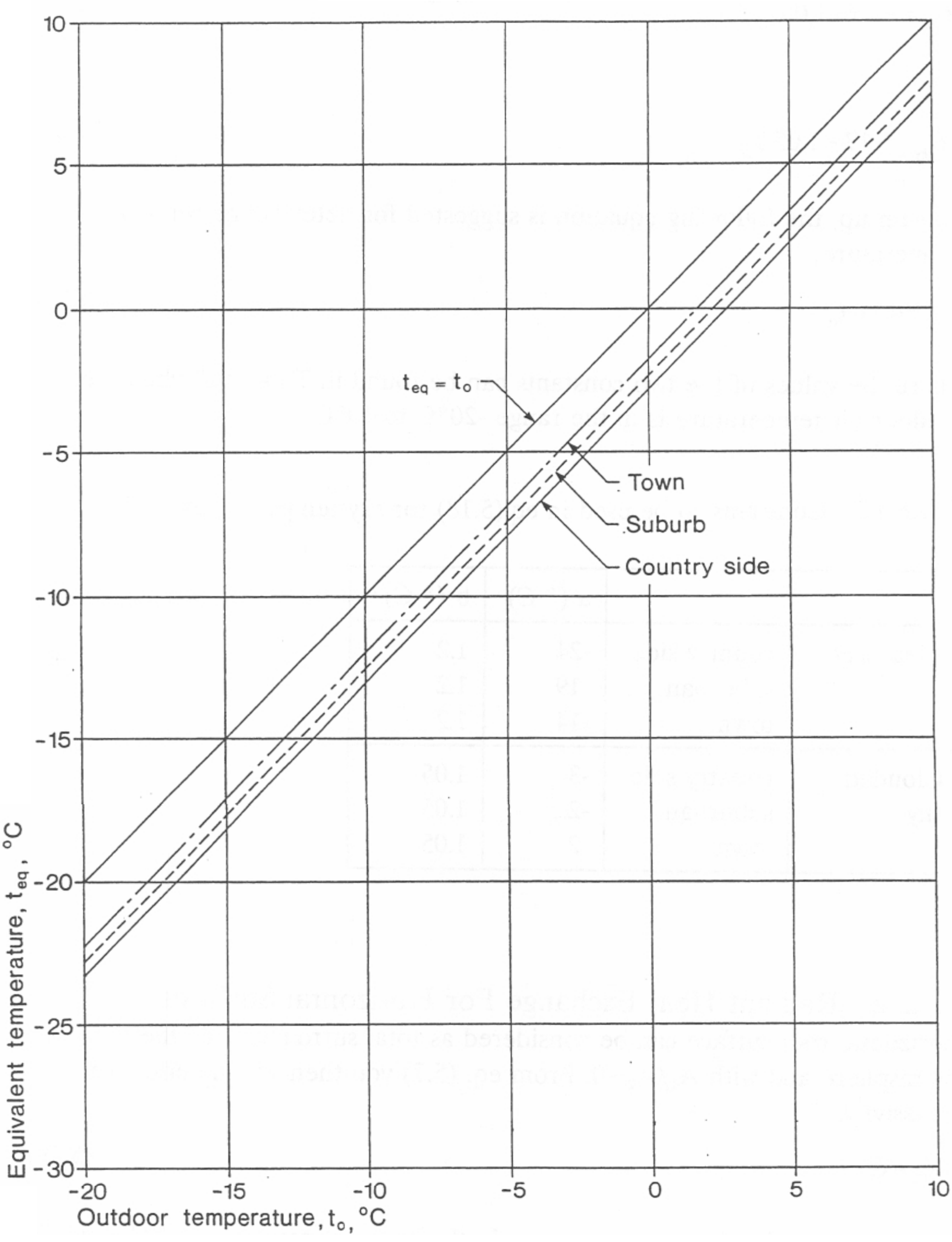


Figure 5.4 *Sky temperatures in nights with clouded sky.*

K to 243 K with a medium value of 253 K. This results in the following linear expression for the outside radiation heat transfer coefficient with an error less than 1%:

$$h_{ro} = 4\epsilon_1 \cdot 5.67 \cdot 10^{-8} \cdot 253^3(1+3(Tm-253)/253) = 3.7\,\epsilon_1(1+0{,}012(T_m-253)) \quad (5.20)$$

A constant value of:

$$h_{ro} = 3.7\,\epsilon_1 \quad (5.21)$$

can be used in the range 248 K < T_m < 258 K with an error less than 9-10% and in the range 261 K < T_m < 271 K with an error less than 14-15%.

With clouded sky and an assumed outdoor temperature range of zero to -10 °C, the sky temperature will be -3 °C to -13 °C and the mean temperature will have a medium value of about 267 K. This gives the following linear expression for h_{ro} valid for 261 K < T_m < 271 K:

$$h_{ro} = 4.3\,\epsilon_1\,(1 + 0{,}011\,(T_m - 267)) \quad (5.22)$$

For the same temperature range the constant value

$$h_{ro} = 4.3\,\epsilon_1 \quad (5.23)$$

can be used with an error less than 10%.

5.2.2.3 Radiant Heat Exchange For Vertical Surfaces.

A vertical surface can be considered as a part of an enclosure together with a half sky hemisphere and the ground. Assuming a view factor of 0.5 for the sky as well as for the ground you can derive a equvalent radiant temperature $T_{r,eq}$ for the surroundings determined by:

$$T_{r,eq} \sim (T_{sky} + T_g)/2 \quad (5.24a)$$

where T_g is the ground temperature. Assuming the ground temperature to be close to the outdoor temperature, eq. (5.24a) gives results close to what was found by Brown (1956) and which is shown on figure 5.5.

For an outdoor temperature in the range of zero to -20 °C, the equivalent radiant temperature will approximately be in the range of -10 °C to -30 °C. Then t_m will be in the range of -5 to -5 °C with a medium value of - 5° C. This gives the following linear expression for h_{ro}:
A constant value of

$$h_{ro} = 3.9\,\epsilon_1 \,(1 + 0{,}012\,(T_m - 259)) \tag{5.25}$$

A constant value of

$$h_{ro} = 3.9\,\epsilon_1 \tag{5.26}$$

can be used with an error less than 10%.

With clouded sky the values of h_{ro} given by eqs. (5.22) and (5.23) can be used.

5.2.2.4 Radiant Heat Exchange For Sloping Surfaces.

For sloping roof surfaces the radiant heat exchange decreases. Petersen (1966) suggest the following reduction factor:

$$f_{red} = (1 + \cos\alpha)/2 \tag{5.27}$$

where α is the angle between the surface and the horizontal. The radiation contribution from the ground can be neglected as the ground temperature will be close to the exterior surface temperature.

By 20 degree roof slope, you then get f_{red} = 0.98 and by 45 degrees you get f_{red} = 0.85. The reduction is thus of minor importance for roofs.

5.3 Convective Heat Transfer Coefficients

By convective heat transfer, the heat energy is transported from one place to another by a moving fluid. In the case discussed here, the moving fluid is air.

The air movement may be induced by buoyancy forces which arise from density differences, which again are caused by temperature differences in the air. This is called free (or natural or buoyant) convection. The air movement may also be caused by external means like fans or atmospheric wind, and this is called forced convection.

The heat transfer from the surface to the moving air takes place in the so-called boundary layer, which is the thin layer closest to the surface, where changes in air velocities and in temperature differences take place.

At the beginning of the surface (by the so-called leading edge), the flow will be laminar, i.e. the flow particles move in layers parallel to the surface, and if

k is the conductivity of the air
ν is the kinematic viscosity of the air

The quantities representing the air properties, i.e. c_p, k, and ν, should principally be evaluated by the average temperature of the moving air. For building surfaces, where the temperature difference will not be larger than 20-30°C, it is sufficiently accurate to evaluate the air properties at the temperature of either the surface or the undisturbed air.

It should be noticed, that the Pr number is a pure fluid property constant. For air with a temperature in the range of -20 °C to 50 °C the magnitude of Pr is 0.70 - 0.72. Therefore a value of Pr = 0.71 will be used in the following.

The equation (5.27) contains some constants and they are found by correlating the equation to experimental data.

An equation similar to eq.(5.28) can be derived for the local Nusselt number and the only difference is that the characteristic length has to be replaced by the distance x from the leading edge to the point in question. There is a definite relationship between the local and the average Nusselt number, which can be found by integrating the local heat transfer coefficient along the whole surface.

5.3.1 Free Convection

By free convection, the flow velocity is determined by the temperature difference so that the Reynold number is not an independent parameter in this case. When using eg.(5.28) for free convection, the Reynold will disappear and you get for the average Nusselt number:

$$Nu_{av} = f(Gr_L, Pr) \tag{5.29}$$

The flow will always be laminar in the beginning, but a transition to turbulent flow may take place. It takes place if instabilities in the flow are not damped sufficiently, and this depends on the ratio between the buoyant and the viscous forces in the flow, which again can be expressed by the product of the Grashof and the Prandtl number. The transition value of this product depends on the direction of the direction of the surface as well as of the heat flux compared to the direction of the gravity.

It should be mentioned that the air velocities by free convection on building surfaces will not be much higher than 1 m/s, and that a convection heat transfer coefficient much larger than 4 W/m^2K should not be expected. By

For large values of $Gr_L Pr$, for instance a value of 10^{11}, the laminar region can be neglected, and the following equations can be used with an error smaller than 5%:

$$Nu_{av} = 0.13(Gr_L Pr)^{1/3} \quad (5.35)$$

and

$$h_c = 1.5(T)^{1/} \quad (5.36)$$

For the local values you get:

$$Nu_X = 0.13(Gr_X Pr)^{1/} \quad (5.37)$$

and

$$h_{cx} = 1.5 (\Delta T)^{1/3} \quad (5.38)$$

A survey of the equations for vertical surfaces is given in table 5.3.

5.3.1.2 Inclined Surfaces. By inclined surfaces the direction of the heat flux has to be taken into account as it influences significantly the magnitude of the heat transfer coefficient. In that connection it is convenient to consider the components of the buoyancy forces normal and parallel to the surface. If the normal component has direction towards the surface it will maintain the boundary layer. If having a direction away from the surface, separation of the boundary layer from the surface will occur, where parcels of air moving away from the surface continuously will be replaced by ambient air, which again results in an increased heat transfer.

The component parallel to the surface is in any case reduced with a factor $\cos\Theta$ compared to its value by vertical surfaces, and where Θ is the angle between the surface direction and the vertical. This gives a similar reduction in the boundary layer velocities and thereby in the convection heat transfer coefficient, as long as the boundary layer follows the surface.

By a heated bottom surface or a cooled top surface, in practice corresponding to a heated ceiling or a supercooled roof, the normal component of the buoyancy force acts towards the surface, and the surface guides the air movements in the boundary layer so that you mainly will have laminar flow. Experimental data confirm, that the equations used for the

Table 5.4 Natural convection on inclined surfaces, isothermal or uniformly heated (Churchill, 1990a, Kreith and Bohn, 1993).

	Laminar regime 1)	Turbulent regime 2)
COLD TOP SURFACE, WARM BOTTOM SURFACE 3)		
Nusselt no:		
Local	$Nu_x = 0.44\ (Gr \cos\theta\ Pr)^{1/4}$	No fully developed turbulence
Average	$Nu_{av} = 0.59\ (Gr_L \cos\theta\ Pr)^{1/4}$	has been measured
Heat transfer coefficient:		
Local	$h_{cx} = 1.1\ (\Delta T \cos\theta/x)^{1/4}$	
Average	$h_{car} = 1.5\ (\Delta T \cos\theta/L)^{1/4}$	
WARM TOP SURFACE COLD BOTTOM SURFACE 4)		
Nusselt no:		
Local	$Nu_x = 0.44\ (Gr_x \cos\theta\ Pr)^{1/4}$	$Nu_x = 0.13\ (Gr \cos\theta\ Pr)^{1/3}$ for $\theta < 45°$ $Nu_x = 0.13\ (Gr \sin\theta\ Pr)^{1/3}$ for $\theta > 45°$
Average	$Nu_{av} = 0.59\ (Gr_L \cos\theta\ Pr)^{1/4}$	$Nu_{av} = 0.13\ (Gr_L \cos\theta\ Pr)^{1/3} - 25.0 \cos\theta$ for $\theta < 45°$ $Nu_{av} = 0.13\ (Gr_L \sin\theta\ Pr)^{1/3} - 130 \sin\theta + 105 \cos\theta$ for $\theta > 45°$
Heat transfer coefficient:		
Local	$h_{cx} = 1.1\ (\Delta T \cos\theta/x)^{1/4}$	$h_{cx} = 1.5\ \Delta T \cos\theta$ for $\theta < 45°$ $h_{cx} = 1.5\ \Delta T \sin\theta$ for $\theta > 45°$
Average	$h_{cav} = 1.5\ (\Delta T \cos\theta/L)^{1/4}$	$h_{cx} = 1.5\ \Delta T \cos\theta - 0.6 \cos\theta/L$ for $\theta < 45°$ $h_{cx} = 1.5\ \Delta T \cos\theta - 3.1 \cos\theta/L + 2.5 \sin\theta/L$ for $\theta > 45°$

1) θ = angle between surface and vertical
2) Valid for $\theta < 87°$
3) e.g. cold floor or warm ceiling
4) e.g. warm floor or cold ceiling
5) Transition takes place for $Gr \cos\theta Pr \sim 10^5\ e^{9\cos\theta}$

indicates that a laminar as well as a turbulent regime has to be considered. A survey of equations of interest is shown in table 5.5.

Table 5.5 Natural convection for horizontal surfaces, isothermal or uniformly heated. (Incropera and De Witt, 1990, Churchill, 1990a).

	Laminar regime 2)	Transition $Gr_x\,Pr$	Turbulent regime
COLD TOP SURFACE, WARM BOTTOM SURFACE 1)		10^{10}	
Nusselt no:			
Local	$Nu_x = 0.29\,(Gr_x\,Pr)^{1/5}$		No fully developed
Average	$Nu_{av} = 0.49\,(Gr_L\,Pr)^{1/5}$		turbulence has been
Heat transfer coefficient:			measured
Local	$h_{cx} = 0.29\,(\Delta T/x^2)^{1/5}$		
Average	$h_{cav} = 0.49\,(\Delta T/L^2)^{1/5}$		
WARM TOP SURFACE, COLD BOTTOM SURFACE 3)		10^5	
Nusselt no:			
Local	$Nu_x = 0.41\,(Gr_x\,Pr)^{1/5}$		$Nu_x = 0.12\,(Gr_x\,Pr)^{1/3}$
Average	$Nu_{av} = 0.70\,(Gr_L\,Pr)^{1/5}$		$Nu_{av} = 0.12\,(Gr_L\,Pr)^{1/3}$
Heat transfer coefficient:			
Local	$h_{cx} = 0.41\,(\Delta T/x^2)^{1/5}$		$h_{cx} = 1.5\,\Delta T^{1/3}$
Average	$h_{cav} = 0.70\,(\Delta T/L^2)^{1/5}$		$h_{cav} = 1.5\,\Delta T^{1/3}$

1) Finite surface with escape possibilities at the edges, e.g. supercooled roof
2) L = area/perimeter
3) e.g. warm roof, heated floor, or cooled ceiling

Table 5.6 Forced convection on flat surface parallel to the flow. (Incropera and De Witt, 1990, Mills, 1992, Wong 1977).

	Laminar regime	Transition Re_x	Turbulent regime
ISOTERMAL		$2 \cdot 10^5$ - $5 \cdot 10^5$	
Nusselt no:			
Local	$Nu_x = 0.33\ Re_x^{1/2}\ Pr^{1/3}$		$Nu_x = 0.029\ Re_x^{0.8}\ Pr^{1/3}$
Average	$Nu_{av} = 0.66\ Re_L^{1/2}\ Pr^{1/3}$		$Nu_{av} = 0.036\ (Re_L^{0.8} - 23000)\ Pr^{1/3}$
Heat transfer coefficient:			
Local	$h_{cx} = 1.9\ (v/x)^{1/2}$		$h_{cx} = 4.6\ v^{0.8}\ x^{0.2}$
Average	$h_{cav} = 3.7\ (v/L)^{1/2}$		$h_{cav} = 5.7\ v^{0.8}\ L^{-0.2} - 18.2/L$
UNIFORMLY HEATED		$\sim 5 \cdot 10^6$	
Nusselt no:			
Local	$Nu_x = 0.45\ Re_x^{1/2}\ Pr^{1/3}$		$Nu_x = 0.031\ Re_x^{0.8}\ Pr^{1/3}$
Average	$Nu_{av} = 0.68\ Re_L^{1/2}\ Pr^{1/3}$		$Nu_{av} = 0.037\ (Re_L^{0.8} - 23000)\ Pr^{1/3}$
Heat transfer coefficient:			
Local	$h_{cx} = 2.6\ (v/x)^{1/2}$		$h_{cx} = 4.9\ v^{0.8}\ x^{-0.2}$
Average	$h_{cav} = 3.8\ (v/L)^{1/2}$		$h_{cav} = 5.9\ v^{0.8}\ L^{-0.2} - 18.7/L$

5.3.3 Combined Free and Forced Convection

In the discussion of forced convection, it has been assumed that free convection did not occur. This is an idealization, as any heat transfer process requires a temperature gradient and thereby density differences when fluids are involved, and this results in free convection. Likewise there will usually be some air movements in a room and around a building due to mechanical ventilation, windforces, etc. so that forced convection may occur in connection with free convection. Therefore, an interaction between free and forced convection has to be considered in order to find out when free or forced convection can be neglected, and when both of them have to be taken into account.

The combined effect of free and forced convection is strongly influenced by the direction of the two contributions relative to each other. They may have the same direction (assisting flow, or have opposite direction (opposing flow), or be perpendicular to each other (transverse flow). These directions may also influence on the start of turbulence. Assisting flow can delay the start whereas opposing flow can promote it.

The combined effect of free and forced convection can be found from the following superposition rule:

$$(Nu_{com})^n = (Nu_{free})^n \pm (Nu_{forc})^n \qquad (5.43)$$

where the exponent n varies according to the specific case to be combined

A theoretical analysis shows that the magnitude of the ratio Gr_L/Re_L^2, representing the ratio between the buoyancy and the inertia forces, can give a qualitative indication on whether one of the contributions can be neglected or both of them have to be taken into consideration.

5.3.3.1 Vertical Surfaces. Laminar Flow.

For laminar, combined convection, experimental data indicate that the best correlation is obtained with an exponent n=3 (Churchill 1990b) in the equation (5.43), so that you get:

$$(Nu_{com})^3 = (Nu_{forc})^3 \pm (Nu_{free})^3 \qquad (5.44)$$

5.3.3.2 Vertical Surfaces. Turbulent Flow. No correlating equation has been found. According to Incropera and Witt (1990) the contribution from free convection can usually be neglected, when the forced flow is turbulent.

5.3.3.3 Horizontal Surfaces. By a forced horizontal flow above a heated horizontal top surface or a cooled bottom surface, the buoyant forces will increase the heat transfer. For a cooled top surface or a heated bottom surface, the heat transfer will be decreased.

For laminar flow, Churchill (1990b) recommend the following equation for the combined heat transfer:

$$(Nu_{com})^{7/2} = (Nu_{forc})^{7/2} \pm (Nu_{free})^{7/2} \tag{5.45}$$

where the positive sign is applicable for heated top and cooled bottom surfaces (e.g. heated floor or cooled ceiling) and the negative sign is for cooled top and heated bottom surface.

If the buoyant forces assist, the free convection can be neglected according to Gebhart (1971) when $Gr_L/Re_L^{5/2} < 0.08$.

If the buoyant forces oppose, eq.(5.45) results in a zero solution for $Nu_{forc} = Nu_{free}$. However, seperation occurs before Nu_{free} exceeds Nu_{forc} (Mills, 1992), so that the equation is only valid for positive Nu_{comb}.

For turbulent flow, no analysis or experimental data are available. Referring to vertical surfaces, it is reasonable to assume that the contribution from free convection is so small, compared to turbulent forced convection, that it can be neglected.

5.4 Interior Building Surfaces

The convection heat transfer coefficients discussed so far are all derived by theoretical considerations supported by experimental data found under controlled laboratory conditions. Under practical conditions, air currents will always appear, which will disturb the convection process, which again usually will increase the convective heat transfer. By interior surfaces, the air currents may for instance be induced by the ventilation system or by vertical and horizontal temperature gradients.

$$Gr_xPr = 10^8 \Delta T x^3 \quad (5.50)$$

The range of this quantity will be from 5x1o[8] and upward, so that laminar as well as turbulent free convection has to be considered.

5.4.2.1 Walls. For walls, it is necessary to distinguish between the inner surfaces of insulated and uninsulated outside walls and of unheated and heated inside walls, respectively. Further, the height of the wall has to be taken into account.

For insulated outside walls up to a height H of about 3 m, the quantity Gr_HPr (x = H) will have a value up to 10^9 resulting in laminar free convection, so that the convection heat transfer coefficient is determined by, cf. table 5.3:

$$h_{ci} = 1.5(\Delta T/H)^{1/4} \quad (5.51)$$

For insulated walls higher than 3 m, the free convection becomes partly turbulent so that you get:

$$h_{ci} = 1.5(\Delta T)^{1/3} - 0.6/H \quad (5.52)$$

or for H > 6 m:

$$h_{ci} \sim 1.5(\Delta T)^{1/3} \quad (5.53)$$

For uninsulated outside walls like glass facades, the temperature difference will be so large that you get values of Gr_HPr of about 10^{10}, so that the convection becomes turbulent and so that eq.(5.53) can be used even for heights down to 2.5 - 3.0 m.

For unheated inside walls the temperature difference will be so small that laminar convection can be assumed even for wall heights above 6 m, so that eq.(5.51) should be used.

For heated inner walls, for instance heated by the sun, temperature differences similar to those for uninsulated outside walls can be expected and even larger, so eq.(5.53) can be used for any height.

5.4.2.2 Cold Floors or Warm Ceilings. In this case, almost no convection should occur. It can therefore be assumed that the convection heat transfer coefficient will be considerably smaller than found by the equation in table 5.5, which is valid for horizontal surfaces where the air can escape along the edges. If a halving is assumed, you get:

Further, they measured the heat transfer from a horizontal free-edged plate with the heated surface facing downward and suspended 1.5 m above the floor. In that case they found a convection heat transfer coefficient about four times larger than the one found for the heated ceiling.

Khalifa and Marshall (1990) made their measurements in a test cell divided into a hot and a cold zone. The wall between the two zones thus functioned as an outside wall. The hot zone had a floor area of 2.95x2.35 m^2 and the room height was 2.05 m. The four walls and the ceiling in this zone were covered with aluminium foil in order to minimize the effect of the longwave radiation exchange. Heat was supplied from the floor or from heat panels covering the wall opposite to the outside wall, or from a radiator either placed along the outside wall or along the inside wall opposite to the outside wall. The difference between inside air temperature and surface temperature was in the range 0.5 -
3.5 °C.

For the floor heating case, they found:

Walls: $h_{ci} = 2.1(\Delta T)$
Floor: $h_{ci} = 2.3(\Delta T)$
Ceiling: $h_{ci} = 2.7(\Delta T)^{0.13}$

For the wall heating case the found:

Outside wall: $2.3(\Delta T)^{0.25}$ =
Wall with heat panels : no result was obtained
Floor: not measured
Ceiling: $\mathbf{h_{ci}} = 3.1(\Delta T)^{0.17}$

For the case with radiator placed opposite to the outside wall, they found:

Outside wall: $h_{ci} = 2.2(\Delta T)^{0.22}$
Wall with radiator: $h_{ci} = 2.4(\Delta T)$
Floor: not measured
Ceiling: $2.8(\Delta T)^{0.14}$

Delaforce et al. (1993) made their measurements in an outside test cell with an inside floor area of 2.03x2.03 m^2 and a height of 2.33 m. The heat was supplied by an air heating system and the temperature difference was in the range of 0.5 - 4.0 °C. When the heat was supplied continuously, they found (with rather large dispersion):

by the circular air flow pattern in the room with air moving upward by the warmer wall and downwarded by the colder one.

The measurements or Delaforce et al. (1993) are also carried out by such temperature differences and surface dimensions that laminar flow is to be expected, but their result are rather dispersed. However, it is of interest to notice their increased values by intermittent heat supply.

Summarizing, it can be stated that the approximate, theoretical solutions for the convection heat transfer coefficients represented by the equations (5.51) - (5.55) are in reasonable good accordance with the available full-scale measurements. Only should the constants in front of the equations be increased by 20 - 25%. Besides, it should be taken into consideration, that the coefficients should be increased further by 15 - 20% by unsymmetrical heat supply.

5.4.5 Recommended Values

The following recommendations are estimates based on the knowledge available.

Outside walls:

insulated, H < 3 m: $h_i = 5.0 + 2.0(\Delta T)^{1/4}$ (5.56)

insulated, 3 m < H < 6 m : $h_i = 5.0 + 2.0(\Delta T)^{1/3} - 0.8/L$ (5.57)

insulated H > 6 m : $h_i = 5.0 + 2.0(\Delta T)^{1/3}$ (5.58)

uninsulated: $h_i = 5.0 + 2.0(\Delta T)^{1/3}$ (5.59)

Inside walls:

unheated: $h_i = 1.0 + 2.0(\Delta T)^{1/4}$ (5.60)

heated: $h_i = 1.0 + 2.0\ (\Delta T)^{1/3}$ (5.61)

Cold floor or warm ceiling:

$h_i = 1.0 + 0.2(\Delta T/L^2)^{0.2}$ (5.62)

Warm floor or cold ceiling:

$h_i = 1.0 + 2.0(\Delta T)^{1/3}$ (5.63)

By unsymmetrical heat supply, the convective part should be increased by 20%.

In any other cases, a more detailed analysis has to be carried out.

Walls, insulated: $\Delta T \sim 5 - 10°C$
uninsulated: $\Delta T \sim 0 - 5°C$
Roofs, insulated: $\Delta T \sim 10 - 15°C$
uninsulated: $\Delta T \sim 5 - 10°C$

The surfaces of the uninsulated walls and roofs are less supercooled due to the larger heat transfer from the inside when a heated building is considered.

In nights with clouded sky, the surface temperature will only be a few degrees above the outdoor air temperature when heated buildings are considered.

It is the situations with low surface temperatures which are of interest (condensation problems, heat losses etc.). Therefore, only the conditions in nights will be discussed in the following.

The equations for determining if the free or the forced convection can be neglected, or if the boundary layer flow is laminar or turbulent, are the equations (5.49) and (5.50). Further you have the following equation for the transition by forced convection:

$$Re_x = vx/\nu \sim 7 \cdot 10^4 vx \qquad (5.64)$$

The values of the constants in these equations do not vary more than about 10% in the temperature range 0 - 30°C

5.5.2.1 Walls. For vertical surfaces, the free convection can be neglected if $Gr_L/Re_L < 0.3$ (cf. section 5.3.3.1) almost independent of, if the free convection assists or opposes the forced convection. The boundary value of 0.3 corresponds to (cf.eq.(5.49)):

$$0.034\Delta TH/v2 < 0.3$$

or

$$v^2 > 0.1\Delta TH$$

For the forced convection you get with the air velocity v > 0.4 m/s and with the wall height H > 3 m :

$$Re_L > 7x10^4 \text{ x } 0.4 \text{ x } 3 \sim 10^5$$

For a supercooled, insulated wall you get, when assuming $\Delta T = 8°C$, that eq.(5.68) is valid for $H/v^2 > 1$, or $v^2 < H$, and this results in $v < 2$ m/s for a 4 m high wall, and $v < 3$ m/s for a 10 m high wall.

For larger air velocities, the free convection can be neglected, and the convection heat transfer coefficient can then be determined by eq.(5.70).

It should be noticed that eqs.(5.68) and (5.69) indicate a zero solution, but experimental data show that the lowest Nu-number is Nu = $0.8Nu_{free}$ (Churchill, 1990).

For a supercooled, uninsulated wall with $\Delta T \sim 4°C$, you get $H/v^2 > 2$ or $v^2 < 0.5H$.

By nights with cloudy sky with $\Delta T \sim 2°C$, you get $H/v^2 > 5$ or $v^2 < 0.2H$.

By transverse flow, i.e. when the wind direction is inclined compared to the building direction, the boundary value for Gr_L/Re_L is about 0.7, cf. section 5.3.3.1, and this means again that free convection can be neglected by air velocities that are 30% smaller than the velocities found above.

5.5.2.2 Horizontal Roofs. For horizontal surfaces, the free convection can be neglected if $Gr_L/Re_L < 0.1$ almost independent of, if the free convection is assisting or opposing the forced convection. This corresponds to :

$$\Delta TH/v^2 < 3$$

or

$$v^2 > 0.3\ \Delta TH$$

Compared to the walls it means an almost doubling of the air velocities before the free convection can be neglected.

For the forced convection, similar considerations can be done as for the walls and you find that the forced convection is only fully laminar by calm weather.

The free convection is strongly dependent on the direction of the heat flux. For a supercooled roof you get from table 5.5:

$$\sim 0.5(\Delta T/L^2)^{0.2} \tag{5.71}$$

Contrary to a cold floor or a heated ceiling, no reduction will be done, because the surface can be considered as finite with escape possibilities at the roof edges.

$$h_c = a + bv \text{ for } v < 5 \text{ m/s} \quad (5.73)$$

$$h_c = cv^d \quad \text{for } v > 5 \text{ m/s} \quad (5.74)$$

A frequently seen reference in textbooks is Jürges (1924), who found the following values for the constants for a rolled surface:

a = 5.8 b = 3.9 c = 7.1 d = 0.78

For polished surfaces he found about 5% smaller values and for more rough surfaces he found about 10% larger values. These values of the constants have on the whole been confirmed by later researchers up through the 1930'es and 1940'es.

Rowley et al (1930a) (cited by Cole and Sturrock, 1977) also examined the influence of the mean temperature (i.e. the average between the air and the surface temperature), and they found a slightly higher convection coefficient by a higher mean temperature.

Rowley et al (1930b) examined the influence of the surface texture. Getting a fair agreement with Jürges (1924) for smooth surfaces, they found an about 25% higher coefficient for more rough surfaces made of plaster, bricks and concrete.

Parmelee and Huebscher (1947) measured on a vertical plate parallel to the flow. They found results similar to those of Jürges (1924). Besides, they found that the length of the surface influenced on the coefficient by giving decreasing values for increasing lengths.

5.5.3.2 Non-parallel flow. A few experiments are carried out with inclined surfaces in wind tunnels. Rowley and Eckley (1933) found with a 0.38x0.38 m^2 surface that an inclination angle between 15° and 90° resulted in a smaller coefficient than by parallel flow, but the value was independent of the angle as long as the air velocity was below 7 m/s. For higher velocities the coefficient was only slightly reduced compared to its value by parallel flow. They concluded that for practical purposes, the coefficient for parallel flow was sufficient accurate also for inclined surfaces.

Sturrock (1971) (cited by Cole and Sturrock,1977) measured the convection coefficient on a 230 mm cube. He found, that the orientation of the surface had a significant influence on the coefficient. Besides, his coefficient values were significantly higher than those found by previous researchers. His results in the velocity range 3 - 10 m/s could be expressed by:

texture is understandable from the point of view that a more rough surface induces an earlier and a higher degree of turbulence, which again increases the heat transfer.

The influence of the length found by Pamelee and Huebscher (1947) is in accordance with what can be found when considering the forced part of eq.(5.68) for sufficiently large air velocities.

The results of Rowley and Eckley, indicating that the inclination angle has almost no influence in a certain range of angles is in good accordance with the results of Tien and Sparrow (1979) mentioned in section 5.3.2.2.

The high coefficient values of Sturrock (1971) can partly be explained by the fact, that he did not eliminate the radiation effects. An other reason might be the different flow pattern around an immersed cube compared to the pattern above a plate mounted flush to one of the surfaces of a wind tunnel.

5.5.4 Full-Scale Measurements

Full-scale measurements are carried out by Sturrock (1971), Ito et al (1972), Nicol (1977), and Sharples (1984). The measurements are carried out in the night to avoid the effect of solar radiation.
Besides, the long wave radiation contribution is subtracted in all the results except those of Sturrock (1971).

Sturrock (1971) made his measurements on a 26 m high building and found for the windward walls:

$$h_{co} = 11.4 + 5.7v_w \qquad (5.76)$$

where v_w is the wind speed measured in the main stream above the roof.

Ito et al (1972) did their measurements on a six storey building and they relates their coefficients to the wind speed measured 8 meter above the roof as well as to the air velocities measured close to the walls. The coefficients related to the wind speed, are rather dependent on the distance from the edges, whereas when related to the surface air velocities, the coefficients are almost independent of this distance. For the surface air velocities they found:

$v_s \sim 0.2v_w$ for windward surfaces and far from edges

$v_s \sim 0.3v_w$ for windward surfaces closer to edges

5.5.4.1 Discussion of Full-scale Measurements. It is natural to compare the full-scale results with the results of the wind tunnel experiments. In this connection it should be pointed out that the velocity in these results is the velocity close to the surface just outside the boundary layer and has nothing to do with the wind speed in the main stream above the roof or 10 m above the terrain.

The work of Ito et al is the most comprehensive of the four full-scale experiments discussed here. His surface air velocities are consistent with what you can get when using relevant pressure coefficients in eq.(5.42). His expression for the air velocities by surfaces far from edges thus corresponds to a pressure coefficient of about 0.95. His expression for the convection coefficient is in fair accordance with the wind tunnel experiments. Only, the temperature difference between the surface and the ambient air is missing. Therefore, it is not possible to evaluate his constant a ~ 6 W/m_2K, which is determined by this temperature difference.

The results of Nicol (1972) are likewise in reasonable accordance with the wind tunnel experiments, when taking into account that for low-rise buildings, you have a pressure coefficient of 0.5-0.6 on the windward side. This gives a surface air velocity of $v_s \sim 0.7 v_r$.

In the eq.(5.76) of Sturrock (1971), the velocity is the wind speed measured in the main stream above the roof. Therefore, he gets convection coefficients, which are about the double of what is found by the other researchers. One reason for these high values is probably, that he did not eliminate the long wave radiation to the sky.

Sharpies (1984) finds rather small convection coefficients. It seems as if problems have occurred in connection with his velocity measurements.

5.5.5 Recommended Values

It is necessary to distinguish between the radiation and the convection contribution when considering the heat transfer between an exterior surface at its surroundings. The recommendations for the radiation part is given in section 5.5.1. Therefore, only the convection part will be discussed in the following.

It should be noticed that the amount of full-scale measurements on exterior surfaces is rather modest, and measurements on roofs are totally lacking.

As to the wall measurements, the results of Ito et al (1972) seems most reliable, and together with the wind tunnel results, they indicate that the semi-

20% should be added in order to take the uncontrolled effects by full- scale measurements compared with laboratory experiments into account.

For a supercooled, insulated wall, (e.g. by a calm, clear night) the free and forced convection are almost equal, and they opposes each other in principle. But, as the air currents around the building will be rather changing in this case, a certain assisting flow may be assumed, and the expression for free, turbulent convection from table 5.3 will be proposed with an increase of 30%. By further adding 20% as mentioned above, you get:

$$h_{co} \sim 1.2 \times 1.3 \times 1.5(\Delta T)^{1/3} = 2.4(\Delta T)^{1/3} \quad (5.81)$$

For windy weather, the free convection can be neglected, and you get by using eq.(5.70) and eq.(5.80) and adding 20%:

$$h_{co} \sim 1.2 \times 5.0(0.4 + 0.64v) = 2.4 + 3.8v \quad (5.82)$$

This equation can be considered as valid for a very smooth surface like glass. For a more rough one, an increase of about 50% should be considered.

For a supercooled, uninsulated wall an analysis gives the same result as for the insulated wall.

For a warm, insulated and uninsulated wall (e.g. cloudy weather) the two convection contributions assist each other. In calm weather this leads to, when taking a certain forced contribution of about 2.5 W/m^{2}K into account:

$$h_{co} \sim 1.2 \times (2.5^3 + (1.5(\Delta T)^{1/3})^3)^{1/3} \quad (28.0 + 5.9\,T)$$

For windy weather eq.(5.82) can be used unchanged.

Summarizing, the following equations for the convection heat transfer coefficient for exterior walls can be recommended based on the knowledge available:

Smooth walls:

Calm weather, supercooled wall: $h_{co} = 2.4(\Delta T)^{1/3}$

warm wall: $h_{co} = (28 + 6\,T)''$

$1\ m/s < v_s < 7\ m/s$: $= 2.4 + 3.8v$,

$7\ m/s \leq v_s$: $v_s^{0.8}$

Rough walls:

Increase the smooth-values with 50%

5.6 Summary

The surface heat transfer is composed of radiation and convection. This chapter contains a description of the theoretical and practical interior and exterior surface heat transfer coefficients.

The principle of radiative heat transfer between surfaces is described. For interior surfaces emissivity values are suggested. For exterior surfaces sky temperature studies are presented and values suggested. The influence of surface slope is also shown.

The theory of free convection on vertical, inclined and horizontal surfaces is presented. The location of the heat source or warm surface is also discussed. Formulas are presented for both forced and combined free and forced convection.

For interior surfaces a method to combine radiation and convection coefficients into one coefficient is presented. Results of laboratory and full scale measurements are presented. Recommendations are given on values for radiation and convective coefficients for warm and cold ceilings, floors and walls.

For exterior surfaces it is suggested not to combine radiation and convection. Results are presented on wind tunnel and full scale measurements and recommended values are given for walls and roofs.

5.8 References

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6 Solar radiation

This chapter gives a description of solar radiation physics and an overview of how different building energy simulation programs deal with this.

In order to distinguish how the different algorithms for solar radiation will influence the results of calculated incident, transmitted and distributed solar radiation in atrium buildings, calculations with different programs are made and compared for a simple building with a glazed space.

A simplified method to estimate solar energy utilisation in glazed spaces is also presented.

6.1 Incident solar radiation

Full treatment of the effect of solar radiation on buildings considers the surfaces of building components as parts of a thermodynamic system comprising the atmosphere, the ground and any buildings in close proximity.

The solar radiation which is received outside the atmosphere by a surface perpendicular to the direction of radiation, the solar constant, is 1.94 Ly/min (1353 W/m^2). The solar constant varies during the year, depending on the varying distance between the sun and the earth, from 1.979 to 2.02 Ly/min. At perihelion, in the beginning of January, the maximum value is 1400 W/m^2 and at aphelion, in the beginning of July, the minimum value is 1309 W/m^2.

Over a year, the breakdown of the radiation which the earth receives from the sun is approximately as follows (Taesler, 1972: Seller, 1965)

Incident radiation at the limit of the atmosphere	100%
Reflection and scatter towards space	30%
Absorption in the atmosphere	17%
Scattered radiation towards the surface of the earth	22%
Direct radiation towards the surface of the earth	31%

wave radiation. The net outward radiation from the ground surface is the difference between the radiation emitted by the ground and counter-reflection from the atmosphere.

The sum of global radiation and the radiation reflected by the ground and other parts of the environment is called total radiation. All these radiant heat transfers occur in the short wave region. When the entire thermodynamic interaction between the surface of a building and the atmosphere is to be studied, the long wave radiation from the atmosphere, the ground and surrounding surfaces must also be included.

Figure 6.1 illustrates the energy fluxes at the surface of a building. These fluxes consist of a convective component

$$(\quad - \quad)$$

where

h_c = surface heat transfer coefficient (W/m^2

T_a = air temperature (°C)

T_s = surface temperature (°C)

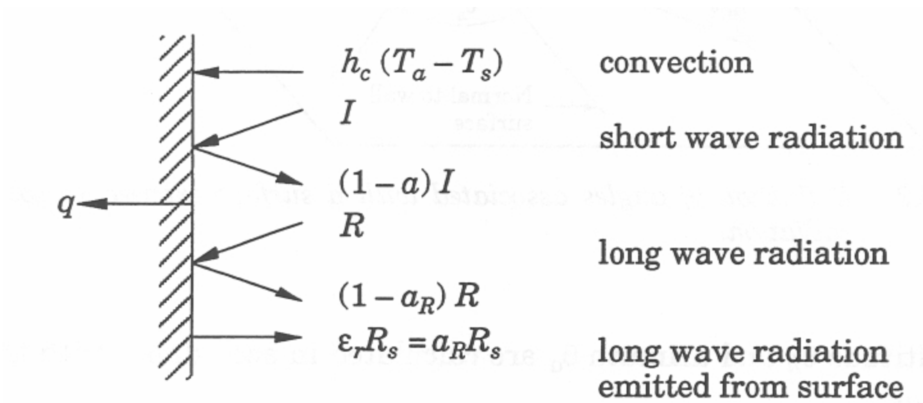

Figure 6.1 Energy fluxes at the surface of a building exposed to solar radiation.

Of the short wave radiation I, the proportion aI is absorbed and the remainder is reflected. The same applies for long wave radiation, i e proportion $a_R R$ is absorbed and the remainder, $(1 - a_R)\,R$ is reflected. Finally, the surface emits the long wave radiation $\varepsilon_r R_s$, where R_s corresponds to the radiation from a black surface with the same temperature T_s as the surface of the building.

By constructing a heat balance for the surface, the following expression is obtained

$$q = h_c\,(T_a - \quad) + aI + \varepsilon_r(R - \quad) \qquad (\mathrm{W/m^2}) \qquad (6.1)$$

$$\theta_d = 0.33281 - 22.984 \cos w_N N_d - 0.34990 \cos 2\, w_N N_d$$
$$- 0.13980 \cos 3\, w_N N_d + 3.7872 \sin w_N N_d$$
$$+ 0.03205 \sin 2\, w_N N_d + 0.07187 \sin 3\, w_N N_d \quad (°) \qquad (6.4)$$

where

$w_N = 2\,\pi / 366$
N_d = day number

Since the solar altitude and azimuth are to be calculated by Equations (6.2) and (6.3) on the basis of civil time, the hour angle must be corrected in accordance with Equation (6.5)

$$\theta_H = 15\,(H - k_m + k_{lH} - 12) \qquad (°) \qquad (6.5)$$

where

H = time (civil time) (h)
k_m = correction in hours in accordance with Equation (6.6) (h)
k_{lH} = correction in hours for the local time deviation from the time meridian (h)

The correction is to be made both in view of the position of the locality in relation to the standard meridian and with regard to the time equation. The time equation gives the difference between true solar time and mean solar time and varies over the year owing to the fact that the orbit of the earth round the sun is elliptical and that this orbit is not in the plane of the equator.

The time equation is given by

$$k_m = 0.0072 \cos w_N N_d - 0.0528 \cos 2\, w_N N_d - 0.0012 \cos 3\, w_N N_d$$
$$- 0.1229 \sin w_N N_d - 0.1565 \sin 2\, w_N N_d - 0.0041 \sin 3\, w_N N_d \quad \text{(h)} \quad (6.6)$$

With the angles defined in Figure 6.2, the intensity of solar radiation on a surface of arbitrary orientation can be written as

$$I = I_{DN} \cos \theta_i \qquad (6.7)$$

where

I_{DN} = intensity of radiation in the direction of radiation (W/m^2)
θ_i = angle between the normal to the surface and the direction of radiation (°)

The coefficient α_r describes Rayleigh scatter and is a function of wavelength in accordance with

$$\alpha_r = \lambda^{-4}\, 0.00816 \tag{6.10}$$

The coefficient α_d is a function of wavelength and is subject to a high degree of variation depending on the turbidity of the atmosphere

$$\alpha_d = \lambda^{-1.3}\, \beta \tag{6.11}$$

where

β = coefficient of turbidity according to Table 6.1.

Table 6.1 Coefficient of turbidity according to Taesler (1985) and Taesler & Andersson (1985).

Month		Month	
Jan	0.040	July	0.065
Feb	0.040	Aug	0.060
March	0.050	Sep	0.055
April	0.060	Oct	0.050
May	0.070	Nov	0.040
June	0.070	Dec	0.040

Optical air mass is a function of solar altitude θ_h and, according to Taesler and Andersson (1985) and Liljequist (1979), can be approximated as follows

$$m = \frac{1}{\sin \theta_h} \qquad \text{when } \theta_h > 10° \tag{6.12}$$

$$m = 1.22\left(\frac{1.0144}{\sin(\theta_h - 1.44)} - 0.49\right) \qquad \text{when } \theta_h \le 10° \tag{6.13}$$

Using the coefficients of absorption in accordance with Equations (6.10) and (6.11), coefficients of turbidity in accordance with Table 6.1 and the optical air mass as determined by Equation (6.12) or (6.13), the intensity of radiation at the surface of the earth is calculated in accordance with Equation (6.9) for an arbitrary wavelength. By integrating (6.9) over the wavelength region of interest, 0.2-10 μm, we obtain the intensity of direct radiation in the direction of the normal as

The intensity of radiation calculated by Equation *(6.14)* is reduced due to absorption by water vapour on its passage through the atmosphere. The absorption is a function according to Liljequist (1979)

$$F = f(w \cdot m) \quad (\mathrm{W/m^2}) \tag{6.15}$$

where

w = quantity of water which can be precipitated (kg/m^2)
m = optical air mass according to Equation (6.12) or (6.13)

The way in which the quantity of precipitable water is calculated on the basis of radio sonde data is described in Liljequist (1979). This reference also describes a simplified method of empirically determining the absorption according to Fowle, in which the quantity of precipitable water is represented by the vapour pressure at the surface. The following expression is given

$$F = 70 + 2.8\, e\, m \quad (\mathrm{W/m^2}) \tag{6.16}$$

where

e = vapour pressure at the surface (mbar)

The absorption according to *(6.16)* is related to the mean distance of the earth from the sun.

The direct radiation in the direction of the normal, corrected for the appropriate distance between the earth and the sun, and with respect to the absorption in water vapour, is obtained from

$$I_{DN} = (I'_{DN} - F) \tag{6.17}$$

where

= correction factor in accordance with Equation (6.18)

The correction factor k_e which takes account of the eccentricity of the earth's orbit around the sun, is obtained from

$$k_e = \frac{1}{1353}(1353 + 45.326 \cos w_N N_d + 0.88018 \cos 2\, w_N N_d - 0.00461 \cos 3\, w_N N_d + 1.8037 \sin w_N N_d + 0.09746 \sin 2\, w_N N_d + 0.18412 \sin 3\, w_N N_d) \tag{6.18}$$

$$I_{dH} = \frac{\eta}{1-\eta} I_{DN} \sin\theta_h \tag{6.25}$$

6.1.2 Short wave radiation from a cloudy sky

In the SOLTIMSYN model (Taesler and Andersson, 1985), solar radiation in conjunction with a cloudy sky is calculated on the basis of synoptic data obtained from meteorological observations.

On the basis of information concerning cloud cover in eights of the sky and the quantities of the different types of cloud, a resultant albedo (reflection) for cloud is calculated by

$$A_c = \frac{N_l A_l + N_m A_m + N_h A_h}{8} \tag{6.26}$$

where

A_c = resultant albedo of cloud
N_l = amount of low cloud, stratus stratocumulus
N_m = amount of middle cloud
N_h = amount of high cloud
A_l = 0.75, albedo for low cloud
A_m = 0.45, albedo for middle cloud
A_h = 0.40, albedo for high cloud

According to Liljequist (1979), global radiation in cloudy weather can be calculated on the assumption that the sky **is** covered by a homogeneous layer of cloud and that no absorption occurs either inside or below the cloud. It is further assumed that the sky is cloudless above the cloud cover.

Of the global radiation which **is** incident on the top of the cloud, the proportion $A_c I_H$ is reflected. The remainder, (1-passes through the cloud and reaches the surface of the earth. Repeated reflection will occur between the surface and the cloud base. The total downward radiation is the global radiation at the surface. The sum is an infinite geometric series, and can be written as

$$I_{H,c} = I_H \frac{1 - A_c}{1 - A_g A_c} \tag{6.27}$$

where

$I_{H,c}$ = global radiation for a cloudy sky (W/m^2)
A_g = albedo of ground

If the effect of the amount of cloud is also included, then, in analogy with Equation (6.28), Equation (6.31) is modified to

$$t_a = \left(\frac{I_{DN}(1 - N_c/8)}{I_o} \right)^{\frac{1}{m}} \tag{6.32}$$

Diffuse solar radiation in the direction towards the sun is treated geometrically in the same way as direct solar radiation, and background radiation is assumed to be wholly diffuse. With the aid of Equation (6.32), diffuse radiation in a direction towards the sun can be written as

$$I_{dN,c} = t_a I_{dH,c} \tag{6.33}$$

where

$I_{dN,c}$ = diffuse solar radiation in the direction towards the sun (W/m^2)

The remaining diffuse radiation is background radiation, given by

$$I_{dB.c} = I_{dH.c} - I_{dN.c} \sin \theta_h \tag{6.34}$$

The diffuse radiation incident on a surface of arbitrary inclination can now be written as the sum of the contributions of diffuse radiation in the direction towards the sun, background radiation and radiation reflected from the ground

$$I_{di,c} = I_{dH,c} t_a \cos \theta_i + I_{dB,c} \frac{1 + \cos \theta_t}{2} + A_g I_{H,c} \frac{1 - \cos \theta_t}{2} \tag{6.35}$$

where the two last terms account for the correction for partially screened horizon and ground surface respectively. See e g Brown and Isfält (1974, pp 97-101)

where

θ_t = inclination of surface to the horizontal plane ($\theta_t = 90 - \theta_\gamma$) (°)

water vapour and carbon dioxide. The emission bands of water vapour are very wide, and atmospheric radiation is to a high degree determined by the temperature in the lower layers of the atmosphere and by the water vapour content of the air. Carbon dioxide has a very wide emission band which is of significance, and radiation in this wavelength interval is approximately equal to that received from an ideal black body.

In e g Liljequist (1979) atmospheric radiation is given by the expression

$$R_A = f(e)\,\sigma T_a^4 \qquad (6.38)$$

where

$f(e)$ = a function which is dependent on the humidity of the atmosphere

T_a = air temperature at ground level (the Stevenson cage) (K)

The average vapour pressure in the atmosphere has a characteristic variation with height above the surface. It is thus possible, in lower layers of the atmosphere, to have the vapour pressure *e* represent the humidity profile of the atmosphere. The contribution made by carbon dioxide is represented by a constant which is included in the function *f (e)*.

In the literature, different expressions are given for the function *f (e)*. The following relationship according to Brunt (1952) has been used

$$R_A = (0.52 + 0.065\sqrt{e})\,\sigma T_a^4 \qquad (6.39)$$

where

e = vapour pressure in mbar at the level of the Stevenson cage

Equation (6.39) is valid only for a cloudless sky and average conditions regarding the variation of temperature and humidity with the height above ground level. The expression is thus not valid when there is e g powerful surface inversion or inversion at a height (Liljequist, 1979).

6.2.2 Long wave radiation from a cloudy sky

When the sky is cloudy, radiation conditions change because the cloud base emits thermal radiation. The cloud base can be regarded as an ideal black body (Liljequist, 1979). In the same way as when the sky is clear, radiation is affected by water vapour and carbon dioxide in the atmosphere.

$$R_{A,Nc} = \frac{N_c}{8}\left((1-k_\lambda)R_G + k_\lambda R_A\right) + \left(1-\frac{N_c}{8}\right)R_A \tag{6.43}$$

where

$R_{A,Nc}$ = long wave radiation from the atmosphere for an amount N_c of cloud (W/m²)

6.2.3 Long wave radiation on inclined surfaces

The total long wave radiation incident on a building surface of arbitrary orientation is then calculated using Equation (6.36), R_A being replaced by in order to take cloud cover into account

$$R = R_{A,Nc}\frac{1+\cos\theta_t}{2} + R_G\frac{1-\cos\theta_t}{2} \tag{6.44}$$

If account is also taken of reflection from the ground, Equations (6.42)—(6.43) are modified. Atmospheric radiation in cloudy weather must be corrected by the term

$$\omega = \frac{1}{1-(1-\varepsilon_g)(1-k_\lambda)} \tag{6.45}$$

Equation (6.44) also includes atmospheric radiation reflected from the ground as expressed by the term R_G. Radiation emitted from the ground when there is broken cloud cover, allowing for atmospheric radiation reflected from the ground, is defined as

$$R_{G,Nc} = \varepsilon_g T_G^4 + (1-\varepsilon_g)R_{A,c} \tag{6.46}$$

Properties for a non-polarised wave can, due to the random fluctuations, be obtained as the average of two polarised waves with their electrical planes perpendicular to each other. The transverse electric *(TE)* wave, also called the parallel component and the transverse magnetic *(TM)* wave, also called the perpendicular component normally are used. The first of these has its electrical plane and the other its magnetic plane parallel to the plane of incidence on an interface between two media.

An optically thick layer has a thickness which is so large compared to the wavelength of the radiation that interference phenomena can be neglected. Commonly used materials in windows, glass or plastic sheets, with a thickness of some millimetres can be treated as optically thick layers. Thin film coatings on glass must often be assumed as optically thin layers as the thickness approaches the wavelength of the radiation, thus interference effects become important. These type of layers are not discussed further.

The refraction index n is defined as the ratio of wave speed in vacuum c_0 to the speed in the media c.

$$n = \frac{c_0}{c} = c_0 \sqrt{\mu\gamma} \tag{6.46}$$

where μ is the magnetic permeability and y the electrical permittivity of the media. This index is valid for media with infinite resistivity and can be used in most cases, e.g. for normal glass we have ≈ 1.5. Note that the refraction index may vary with the wavelength.

To quantify the absorption within a media an extinction coefficient k is used. For a given path length x in a media, the absorption in a single pass through the media **is** given by Bouger's law (also referred to as Beer's law).

$$a = 1 - e^{-k_g} \tag{6.47}$$

For some glass types $k_g \approx$ 23.3 m^{-1} is used and 19.6 m^{-1} is often seen for float glass. In this case no reflection is assumed in the media and according to thermodynamic equilibrium we get the transmission:

$$t = 1 - a \tag{6.48}$$

The gaps between the panes in windows are often filled with air. In some cases a heavy gas is used in a sealed unit to reduce the U value. Air, as well as commonly used gases, can be treated as vacuum when calculating solar radiation through windows and within buildings. Absorption, scattering et cetera in gases are only needed to take into account when looking at the propagation through the atmosphere, thus we can use a refraction index $n = 1$ and an extinction coefficient $k = 0$ for all gases.

$$r_{TM} = \left(\frac{n_1 / \cos\theta_1 - n_2 / \cos\theta_2}{n_1 / \cos\theta_1 + n_2 / \cos\theta_2}\right)^2 \qquad (6.55)$$

With two semi-infinite media there are no multiple reflections nor any absorption at the interface. Thus, according to thermodynamic equilibrium, we get the transmission through the interface.

$$t_{TE} = 1 - r_{TE} \quad \text{and} \quad t_{TM} = 1 - r_{TM} \qquad (6.56)$$

6.3.2 Single panes, optically thick layers

The radiation in an optically thick layer between two semi-infinite media is illustrated in Figure 6.4. The incident radiation from media 1 is partly transmitted to media 2 and partly reflected back. In the layer multiple reflections will occur causing multiple reflected beams back into media 1 and multiple transmitted beams into media 2.

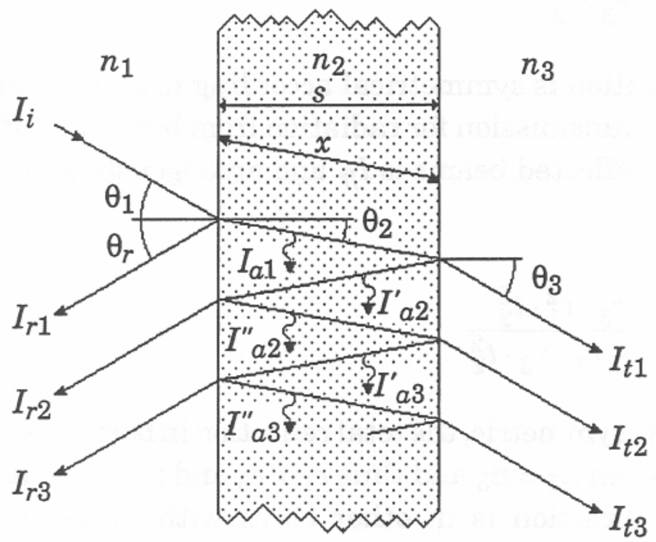

Figure 6.4 The radiation in an optically thick layer between two semi-infinite media.

For a given incident angle Θ_1 at the first surface, Eq 6.49 gives the reflection angle Or and Eq 6.50 the refraction angle Θ_2 which then will be the incident angle at the second surface were the same equation gives the next refraction angle Θ_3. According to Eq 6.49 all waves reflected inside media 2 have the same reflection angle Θ_2, thus all waves transmitted back into media 1 and those transmitted into media 3 have the directions Θ_r and θ_3 respectively.